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Considering Risk Issues in the Uncapacitated Facility Location Problem with Stochastic Components

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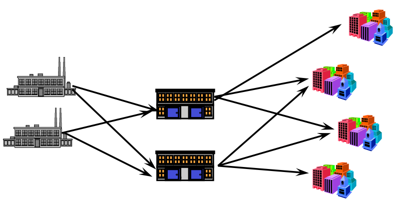
Outline of the Presentation

- Introduction
  - Supply Chain Design
- The Uncapacitated Facility Location Problem (UCFLP)
- UCFLP with stochastic components
- Computational results
- Conclusions
- Directions of future work.



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Introduction

- Design of logistics network
  - ¿Where to locate?
    - Factories
    - Warehouses
    - Sale points/stores
  - With the objective to …
    - Maximize sales
    - Minimize costs
    - Maximize service quality

Introduction

- Design of logistics network
  - Risk of location warehouses
    - Variable costs
    - Variable demands
    - Variable time (traffic)
  - Consider risk during decision making
    - More realistic
    - Better decision

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Mathematical Model of the UFLP

- The Uncapacitated Facility Location Problem (UCFLP)
  - Data
    - * $\mathbb{F}$ = set of potential facilities
    - * $\mathbb{C}$ = set of customers
    - * f_i = fixed cost for opening a facility
    - * c_{ij} = unit cost of supplying customer *i* from facility *j*.

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Mathematical Model of the UFLP

- The Uncapacitated Facility Location Problem (UCFLP)
  - Variables
    - * x_{ij} = fracion of demand of customer *i* served by facility *j*.
    - * y_j = binary variables that indicate if facility *j* is open (1) or close (0).

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Mathematical Model of the UFLP

- The Uncapacitated Facility Location Problem (UCFLP)
  - Model

$$\min_{x,y} Z_{det} = \sum_{i \in \mathbb{F}} f_i y_i + \sum_{i \in \mathbb{F}} \sum_{j \in \mathbb{C}} c_{ij} x_{ij} \qquad (1a)$$

$$\text{s.t.} \quad \sum_{i \in \mathbb{F}} x_{ij} = 1 \qquad , \forall j \in \mathbb{C} \qquad (1b)$$

$$x_{ij} \leq y_i \qquad , \forall i \in \mathbb{F}, j \in \mathbb{C} \qquad (1c)$$

$$0 \leq x_{ij} \leq 1, \; y_i \in \{0,1\} \qquad , i \in \mathbb{F}, j \in \mathbb{C}; \qquad (1d)$$

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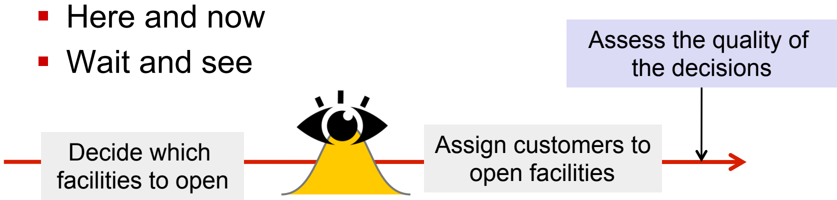
UCFLP - Stochastic Components

- Serving costs are stochastic
  - Cost for travel time
  - Cost for demand
- Two-stage programming Model
  - Here and now
  - Wait and see



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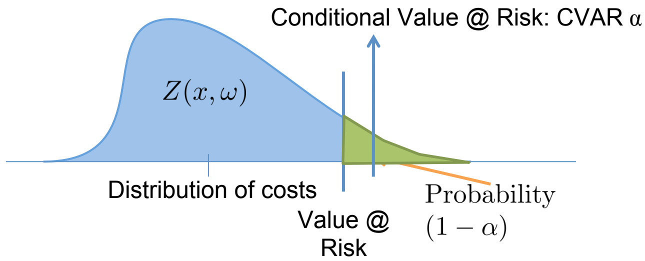
UCFLP - Stochastic Components

- Conditional Value-at-Risk (CVaRα)

$$\mathrm{CVaR}_{\alpha}(W) = E(W|W \geq VaR_{\alpha}(W))$$



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UCFLP - Stochastic Components

- Mean-risk model
  - A mean-risk objective allows to express the decision maker attitude towards risk by fixing a parameter λ to weight the overall expected cost versus risk measure.
- Finite set of scenarios $k \in S$
  - π^k = probability of scenario k
  - ν^k is the amount by which each scenario exceeds the value-at-risk.
  - η is the value-at-risk.

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UCFLP - Stochastic Components

$$\min_{y,x,v,\eta} Z_{stoch} = (1-\lambda)\left\{\sum_{i\in\mathbb{F}} f_i y_i + \sum_{k\in\mathbb{S}} \pi^k \sum_{i\in\mathbb{F}} \sum_{j\in\mathbb{C}} c_{ij}^k x_{ij}^k\right\} + \quad (1a)$$

$$\lambda\left\{\eta + \frac{1}{1-\alpha}\sum_{k\in\mathbb{S}} \pi^k v^k\right\}$$

$$\text{s.t.} \sum_{i\in\mathbb{F}} x_{ij}^k = 1 \quad , j\in\mathbb{C}, k\in\mathbb{S} \quad (1b)$$

$$x_{ij}^k \le y_i \quad , i\in\mathbb{F}, j\in\mathbb{C}, k\in\mathbb{S} \quad (1c)$$

$$v^k \ge \sum_{i\in\mathbb{F}}\sum_{j\in\mathbb{C}} c_{ij}^k x_{ij}^k - \eta \quad , k\in\mathbb{S} \quad (1d)$$

$$x_{ij}^k \ge 0,\ y_i \in \{0,1\} \quad , i\in\mathbb{F}, j\in\mathbb{C}, k\in\mathbb{S} \quad (1e)$$

$$\eta\in\mathbb{R}, v^k \ge 0 \quad , k\in\mathbb{S}, \quad (1f)$$

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Computational Results

- Assume costs have a lognormal distribution
- Each element c_{ij}^k is sampled from lognormal X
  - $E(X) = c_{ij}$
  - $V(X) = \kappa E(X)$
- Scenarios generated by Monte Carlo Simulation
- Tests cases implement in Java using CPLEX 12.1 to solve the model.

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Computational Results

- Results from solving the case B1.1 of the Bilder-Krarup benchmark for different values of λ and κ= 2:

Z^*_{stoch}	$\lvert\mathbb{S}\rvert$	α	λ	Fixed costs	Variable costs	Total cost	Num. Open Facilities
23468,0	1	-	0	9779	13689	23468,0	5
23400,3	200	0,95	0	9779	13621	23400,3	5
21003,2	200	0,95	0,25	11645	12183	23828,4	6
17952,8	200	0,95	0,5	14606	10502	25108,2	7
13669,7	200	0,95	0,75	18861	8738	27599,5	9

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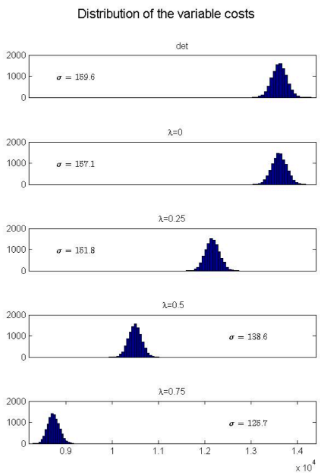
Computational Results

- Histogram of the variable costs for case B1.1 (κ= 2) varying the λ parameter.



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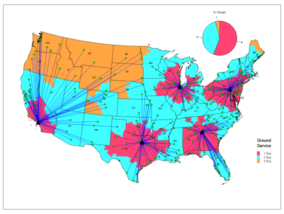
Conclusions

- A variant of the UFLP considering stochastic costs and risk is presented.
- The solutions found could not be obtained by the deterministic version.
- Evaluates the impact of the risk on the decision making.
- Help the decision making.



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Future Research

- Run for a large number of instances
- Evaluate the results obtained
- Simheuristics to solve this problem for large scale instances.
- Stochastic Capacitated Facility Location Problem

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