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From ILS to Hybrid ILS …

and other extensions

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Outline

- Introduction to metaheuristics and ILS
- Some history
  - My story and first applications
- Applications of ILS
- Hybrid ILS and other Extensions
  - Hydrid with other metaheuristics
  - SimILS
  - BiasILS
  - MathILS
  - Bilevel optimization using ILS

Search...

*Levantou-se, e, fiel à regra de que em todas as operações de busca o melhor é começar sempre por uma ponta e avançar com **método e disciplina**, atacou o trabalho pelo extremo de uma fileira de estantes, resolvido a não deixar papel sobre papel sem verificar se, entre o de baixo e o de cima, outro papel não estaria escondido.*

Todos os nomes, José Saramago, Editoral Caminho, 1999

"He stands up and, following the law that in all search operations the best thing is always to start from one point and **advance methodically and with discipline**, he attacks the job from one end of the bookshelf, resolved not to leave any page unturned without checking whether, between the lower and upper one, there is another paper hidden."

All the names, José Saramago

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Optimization Problems

- Combinatorial Optimization problem
  - Given a set of elements $E = \{1, 2, ..., n\}$
  - Set of feasible solutions F
    - Each element of F is a subset of E.
  - Objective function $f(x): F \rightarrow R$.
  - In the minimization version the problem consists in
    - Finding $x^* \in F$, such that $f(x^*) \leq f(x)\ \forall x \in F$.
- Discrete Optimization
- Global Optimization
- Non-smooth Optimization …

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Combinatorial Optimization Problems

- Routing problems
  - Vehicle Routing Problem
  - Heterogeneous Vehicle Routing Problem
- Scheduling Problems
  - Job-shop scheduling problem
  - Parallel machines
- Other …
  - Clique problems
  - Cloud computing

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Combinatorial Optimization Applications

- Applications in many different industries and sectors:
  - **Telecommunications**
  - **Transportation**
  - **Logistics**
  - **Manufacturing**
  - **Marketing**
  - **Health care**
  - Energy
  - Bioinformatics
  - …

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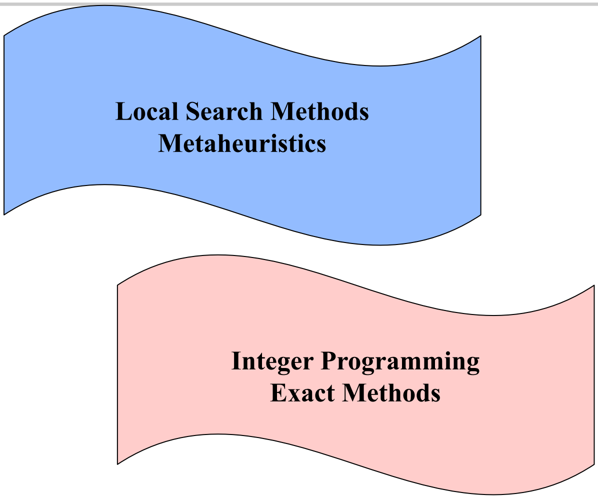
Solution Methods



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Algorithms

- Heuristics
  - Any approximate method build up on the basis of the structural properties or on the characteristics of the problem solution, with reduced complexity with respect to exact methods and providing, in general, good feasible quality solutions, without a forma guarantee of solution quality.
    - * Cost vs. Running Time
- Metaheuristics
  - The process of finding a good solution (eventually the optimum) consists in applying at each step (guiding) a subordinate heuristic which has to be design for each particular problem.
    - * C. Ribeiro [1996]

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Solution Methods

Local Search Methods Metaheuristics

- Tabu Search
- **Iterated Local Search**
- Simulated annealing
- VNS
- Genetic Algorithms
- Evolutionary algorithms
- Ant Colony Optimization
- Scatter Search
- Memetic Algorithms
- Etc… Etc...

Metaheuristics for real problems

- Metaheuristics
  - Highly effective on hard problems
  - Modularity
  - Easy implementation
  - Short updates
  - Robust
  - Able to give good solutions in short time.
- Real Problems
  - Complex problems
  - Rapid changes in reality
  - Need to quick implementation
  - Different aspects in different sectors
  - Need to quick answer and multiple scenarios

Metaheuristics

- Four attributes of Heuristics and Metaheuristics:
  - Accuracy
    - Close to optimal
  - Speed
    - Small computational time
  - Simplicity
    - No parameters adjustment / easy to program
  - Flexibility
    - Easy to adapt to other real problems

Cordeau JF, Gendreau M, Laporte G, Potvin JY, Semet F (2002) A guide to vehicle routing heuristics. J Oper Res Soc 53:512–522.

Local optimization algorithms

- Given a solution, attempts to improve this solution by making local modifications at each iteration.
- **Neighborhood**
  - $N: A \rightarrow 2^A$ subset of feasible solutions
  - Define a local modification, move;
    - the neighborhood of a solution x is the subset of feasible solutions obtained from applying this move to x.
  - Local optima solution x:
    - $c(x) \leq c(y)$ for all $y \in N(x)$
- Search Strategy
  - the name of the metaheuristic come usually from the type of search.

Local optimization algorithms

- Local search
  - 1. Get a initial solution x (current solution). Use a constructive heuristic.
  - 2. Search the neighborhood. While there is an untested neighbor of x:
    - 2.1 Let x' be an untested neighbor of x;
    - 2.2 If c(x')<c (x) set x=x'; (x' is the new current solution)
  - 3. Return x (local optimal solution).

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Local optimization algorithms

- Design of a local optimization algorithm:
  - Obtain an **initial solution**
    - Heuristic
    - Random solution
  - Define the **neighborhood**
    - Specific for each problem
  - How to **search the neighborhood**
    - Complete search
    - First improvement

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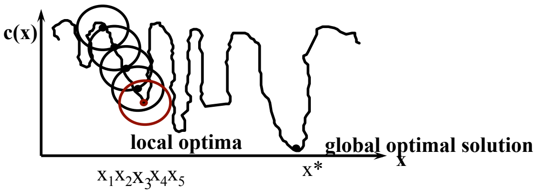
Local optimization algorithms

- Comments
  - The search stops at the first local optimum solution with respect to the neighborhood N.
  - The final solution highly depends on the initial solution and on the neighborhood.
  - No way back out of unattractive local optima...



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Multi-Start

- Iterative improvement or hill-descending
  - 1. Get a initial random solution x.
  - 2. Run an **local optimization** (output x)
  - 3. If cost(x)<cost(x_{best}) set x_{best}=x;
  - 4. If the stop criteria is not verified, go back to step 1.
  - 5. Output the best solution found.

    * Comments
      – Successive repetition of local improvement.
      – Easy to implement.
      – Random solutions may be very bad.

Iterated Local Search

- A Local Search Method…
  - Single chain on search
- Search on the space of local optimal solutions
- Combines local optimization with a big transition/large step/perturbation.
  - Perturbation should not be easily undone by the local search
  - Most important aspect of the ILS
- Able to make large changes at any stage of the algorithm.

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Iterated Local Search

- Get an **initial solution** x;
  - * Heuristic method or a random solution.
  - * **local optimization** method
- For a certain number of iterations:
  - **Perturbation Step**
    - * method that makes a large modification based in optimization and on the structure of the solution x, resulting in x'.
  - Small-steps
    - * **local optimization** method, initial solution x'; final solution x''.
  - **Perform an accept/reject test**
    - * accept all solutions, accept with a certain probability or accept only if it is a better solution.
    - * If x'' is accepted, then x = x''.
- Return the best solution found.

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Iterated Local Search

- A simple implementation…
  - **Generate Initial Solution**
    - Greedy Heuristic
  - **Local Search Method**
    - First improvement local search
    - Definition of neighborhood
  - **Perturbation Method**
    - One move of a high level neighborhood
  - **Acceptance Criteria**
    - Accept if a better solution is found

Often leads to very good performance

Only requires few lines of additional code

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Iterated Local Search

- **Generate Initial Solution**
  - Randomized Greedy Heuristic
  - Random solution
- **Acceptance Criteria**
  - Better
  - Random Walk
  - Simulated Annealing type
  - Restart
- **Local Search Method**
  - Local search
  - Tabu search
- **Perturbation**
  - Higher level of neighborhood
  - Strength of the perturbation
    - Big/small
  - Adaptive memory
  - Modify input data
  - Optimized perturbation

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Iterated Local Search

- Improving ILS
  - **Relationship between local search and perturbation.**
    - * Perturbation must lead to a new region of the solution space that cannot be reached by a local search method.
    - * Perturbation should not be easily undo by the local search.
  - Perturbation can incorporate problem-specific information.
    - * As for example optimization methods
    - * Destruction and construction approach
  - A good perturbation transforms one excellent solution into a excellent starting point to a local search.
  - Local search method must be fast.
  - Complexity must be added progressively and in a modular way.

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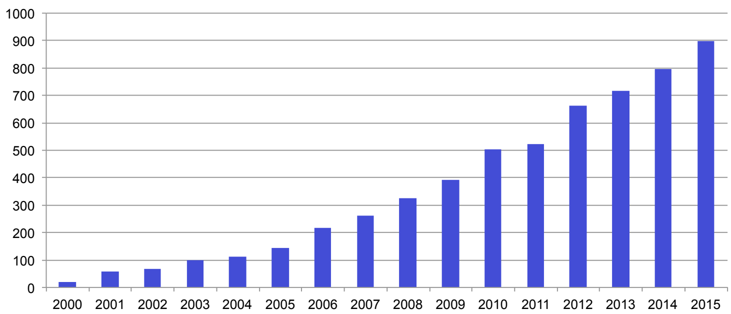
Iterated Local Search

- Google Scholar's number of publications using **"Iterated Local Search"**
  - About 6000 publications



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Iterated Local Search

- ILS applied to Complex and large-scale real problems

Accuracy	• Complex problems. • Large scale problems.
Speed	• Fast answer. • Analysis of several scenarios.
Flexibility	• Fast changes. • Different constraints in different areas.
Simplicity	• Need of fast implementation

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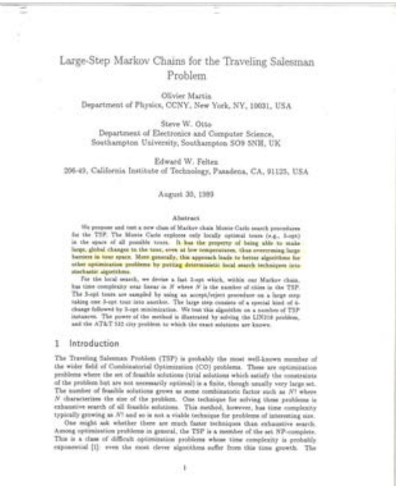
Some History…

- First time I heard about ILS…
  - Not named ILS yet!
  - **"Large Step Markov Chains for the Traveling Salesman Problem"**
  - Olivier Martin, Steve Otto, Edward Felten (1992) Operations Research Letters 11:219-224



Large-Step Markov Chains for the Traveling Salesman Problem

Olivier Martin
Department of Physics, CCNY, New York, NY, 10031, USA

Steve W. Otto
Department of Electronics and Computer Science,
Southampton University, Southampton SO9 5NH, UK

Edward W. Felten
206-49, California Institute of Technology, Pasadena, CA, 91125, USA

August 30, 1989

Abstract

We propose and test a new class of Markov chain Monte Carlo search procedures for the TSP. The Monte Carlo explores only locally optimal tours (e.g., 3-opt) in the space of all possible tours. It has the property of being able to make large, global changes to the tour, even at low temperatures, thus overcoming large barriers in tour space. More generally, this approach leads to better algorithms for other optimization problems by putting deterministic local search techniques into stochastic algorithms.

For the local search, we devise a fast 3-opt which, within our Markov chain, has time complexity near linear in N where N is the number of cities in the TSP. The 3-opt tours are sampled by using an accept/reject procedure on a large step taking one 3-opt tour into another. The large step consists of a special kind of 4-change followed by 3-opt minimization. We test this algorithm on a number of TSP instances. The power of the method is illustrated by solving the LIN318 problem, and the AT&T 532 city problem to which the exact solutions are known.

1 Introduction

The Traveling Salesman Problem (TSP) is probably the most well-known member of the wider field of Combinatorial Optimization (CO) problems. These are optimization problems where the set of feasible solutions (trial solutions which satisfy the constraints of the problem but are not necessarily optimal) is a finite, though usually very large set. The number of feasible solutions grows as some combinatoric factor such as N! where N characterizes the size of the problem. One technique for solving these problems is exhaustive search of all feasible solutions. This method, however, has time complexity typically growing as N! and so is not a viable technique for problems of interesting size.

One might ask whether there are much faster techniques than exhaustive search. Among optimization problems in general, the TSP is a member of the set NP-complete. This is a class of difficult optimization problems whose time complexity is probably exponential [1]: even the most clever algorithms suffer from this time growth. The

1

Some History…

- "Large Step Markov Chains for the Traveling Salesman Problem"
  - Lin-Kernighan local search heuristics
  - The heuristic combines "this good feature of the local search method with Markov chains so to produce a more powerful type of Monte Carlo procedure than the standard simulated annealing method".
  - Iterated Local opt
    - Local search 3-opt
    - Perturbation (kick move): a random 4-change move and double-bridge

Some History…

- I got interested in the method proposed by O. Martin, S. Otto, E. Felten …
- Apply it to Job-shop Scheduling problem in my Ph.D. thesis (1993)
  - Lourenço H.R. (1995), Job-Shop Scheduling: computational study of local search and large-step optimization methods. European Journal of Operational Research 83(2): 347-364.

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Some History…

- Job-Shop Scheduling: computational study of local search and large-step optimization methods (1995)
  - Initial solution
    - Priority Dispatching Rule by Adams, Balas and Zawack (1988)
  - Local search method
    - Local Search Methods 2-opt by Laarhoven, Aarts and Lenstra (1992)
    - Simulated Annealing
  - Perturbation phase
    - Solving two random machines to optimality One Machine Scheduling Problem by Carlier & Pinson (1989)
    - Solving two random machines heuristically with Early-Late Algorithm.

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Some History…

- Job-Shop Scheduling Problem (1995) and (1996)
  - Local search method
    - Local Search Methods 2-opt by Laarhoven, A…)
    - Simulated Annealing
    - Tabu Search
  - Perturbation phase
    - Solving two random machines to optimality One … Problem by Carlier & Pinson (1989)
    - Solving two random machines heuristically with Early-Late Algorithm.

Hybrid Metaheuristic

Matheuristic

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Some History…

- "Applying **Iterated Local Search** to the Perturbation Flow Shop Problem", **Thomas Stützle** (1998) technical report, TU Darmstadt.
  - Initial solution: Nawaz, Enscore and Ham heuristic.
  - Local search: insertion
  - Perturbation: swap and exchange
- Thomas and I first met at
  - **Metaheuristics International Conference**
  - Angra dos Reis, 1999

Some History

- Many names
  - Large Step Markov Chains
  - Iterated Local Search
  - Chained Local Optimization
  - Iterated Descent
  - Iterated Lin-Kernighan
  - Local Search with Perturbation
  - Iterated Greedy Algorithm
  - …
- Previous works by Baxter (1981), Baum (1986), Lin and Kernighan (1973)…

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ILS Applications

- Traveling Salesman Problem
- Maximum Cut-Clique Problem
- An ILS algorithm for water distribution network design optimization
  - Annelies De Corte, Kenneth Sörensen (2016) Networks
- Many others applications…
  - Scheduling
  - Routing
  - Assignment
  - Etc.

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Traveling Salesman Problem

- Traveling Salesman Problem
  - Given a number of cities and the costs (distances) of traveling from any city to any other city…
  - What is the least-cost round-trip route that visits each city exactly once and then returns to the starting city?
  - http://www.math.uwaterloo.ca/tsp/

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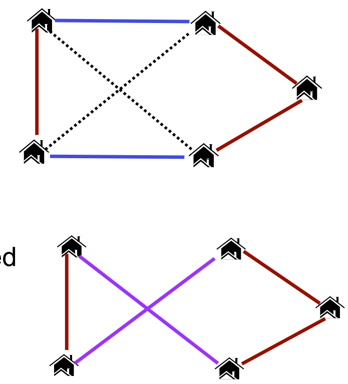
Traveling Salesman Problem

- Generate Initial Solution
  - Constructive Heuristic: nearest neighbor, insertion heuristic
- Local Search Method
  - 2-opt/3-opt Neighborhood
- Perturbation Method
  - 4-opt move (double-bridge)
- Acceptance Criteria
  - Accept only if the best solution improved



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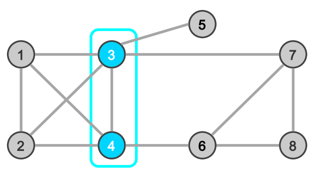
Maximum Cut-Clique Problem

- Given a clique C, its edge neighborhood (cut-clique) is defined by the set of edges $E'(C)= \{(i,j) \in E : i \in C \text{ and } j \in V \setminus C \}$, and $|E'(C)|$ is its size. Denote $N(i)=\{j\in V:(i,j)\in E\}$.
- Maximum Cut-Clique
  - Maximum edge neighborhood clique

$\max |E'(C)| = 7$



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Maximum Cut-Clique Problem

- Perturbation
  - Select randomly node
  - Build the clique with all nodes in the previous clique and fully connected to this node

    Set $C \leftarrow [C \cap N(i)] \cup \{i\}$;
    Set $U \leftarrow \varnothing$ and $C' \leftarrow C$;
- Local Optimization
  - Add, Swap and Aspiration moves

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Computational results:

Intel Core i7-2600 with 3.40GHz and 8GB RAM; using CPLEX 11.2

The ---- symbol means that CPLEX was not able to read and preprocess the model in one hour

time in seconds

Instance	$\|V\|$	$\|E\|$	d	MC problem		MCC Problem		
				$\omega(G)$	$\|N(C)\|$	$\|C\|$	$\|N(C)\|$	time
d1-RTN	2418	9317	0.0032	10	195	8	1273	605.11
d3-RTN	4755	26943	0.0024	18	1097	----	----	----
d7-RTN	6511	44615	0.0021	18	1576	----	----	----
d15-RTN	7965	62136	0.0020	18	1979	----	----	----
d30-RTN	10101	91803	0.0018	21	13099	----	----	---
d66-RTN	13308	148035	0.0017	----	----	----	----	----
c-fat200-1	200	1534	0.077	12	72	9	81	0.05
c-fat200-2	200	3235	0.163	24	264	17	306	0.09
c-fat200-5	200	8473	0.426	58	1682	44	1892	0.05
c-fat500-1	500	4459	0.036	14	98	11	110	0.76
c-fat500-2	500	9139	0.073	26	338	19	380	0.80
c-fat500-5	500	23191	0.186	64	2048	48	2304	0.83
c-fat500-10	500	46627	0.374	126	7938	94	8930	0.58

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Computational results of the ILS:

Intel Core i7-2600 with 3.40GHz and 8GB RAM

100 runs

time in seconds

Instance	| V |	| E |	d	MCC Problem | C |	| N(C) |	time
d1-RTN	2418	9317	0.0032	8	1273	0.1762
d3-RTN	4755	26943	0.0024	12	3526	0.4743
d7-RTN	6511	44615	0.0021	15	5656	0.6777
d15-RTN	7965	62136	0.0020	16	7772	0.8757
d30-RTN	10101	91803	0.0018	21	13099	1.1317
d66-RTN	13308	148035	0.0017	28	22379	1.4081
c-fat200-1	200	1534	0.077	9	81	0.1385
c-fat200-2	200	3235	0.163	17	306	0.0866
c-fat200-5	200	8473	0.426	44	1892	0.0664
c-fat500-1	500	4459	0.036	11	110	0.5451
c-fat500-2	500	9139	0.073	19	380	0.3595
c-fat500-5	500	23191	0.186	48	2304	0.2381
c-fat500-10	500	46627	0.374	94	8930	0.2111

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Market Basket Analysis

- The main objective is to analyze large dataset of store transactions
- Obtain relevant insights to do a better planning of the Marketing strategies and operations.
  - Product placement
  - Optimal product-line offering
  - Personalized marketing campaigns
  - Product promotions

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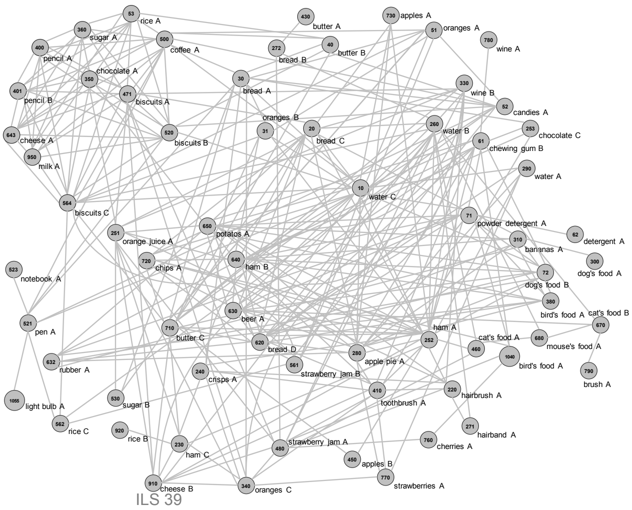
Market Basket Analysis

nodes:
products in a store

edges:
represent pairs of products (*i*,*j*) bought together by a customer on a given purchase visit to the store



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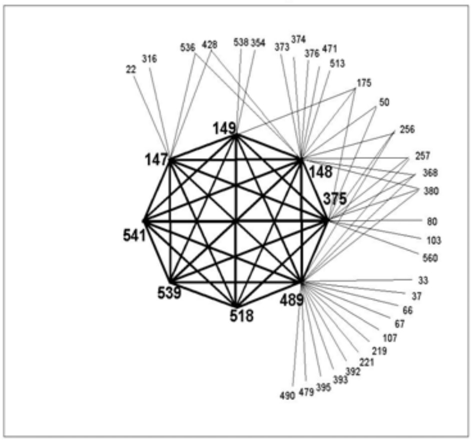
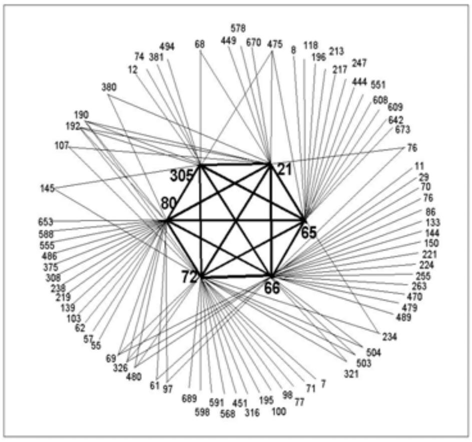
Market Basket Analysis





maximum cardinality clique **maximum cut-clique**

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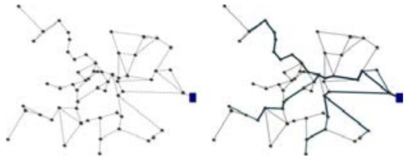
The water distribution network design

- Design a water distribution networks
- Find the least-cost pipe configuration that satisfies hydraulic laws and customer requirements, using a limited set of available pipe types.
- The aim is to find the best diameter for every pipe in network.



An iterated local search algorithm for water distribution network design optimization
Annelies De Corte, Kenneth Sörensen (2016) Networks
Volume 67, Issue 3, pages 187-198, 18 FEB 2016 DOI: 10.1002/net.21673
http://onlinelibrary.wiley.com/doi/10.1002/net.21673/full#net21673-fig-0001

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The water distribution network design

- Generate Initial Solution
  - highestCost
    - * sets all pipe diameters to the largest available diameter in the set of commercially available pipe types.
  - lowCost
- Local Search Method
  - Move: decrease (reduces the pipe diameter of one)
- Perturbation Method
  - A fixed percentage of all pipes is randomly selected.
  - Increases the diameters of these pipes with one size.

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The water distribution network design

- The algorithm is also applied to a set of HydroGen test networks.
- The algorithm was tested on 50 different networks.
- ILS found the optimal solution
  - When applied to the well-known and widely used benchmark networks: the New York City Tunnels network, the Hanoi network, and the two loop network.

Iterated Local Search

- Main References

  - ***Lourenço H.R***., Martin O. and Stützle T. (2010), Iterated Local Search: Framework and Applications. In Handbook of Metaheuristics, 2nd. Edition. Vol.146. M. Gendreau and J.Y. Potvin (eds.), Kluwer Academic Publishers, International Series in Operations Research & Management Science, pp. 363-397.
  - ***Lourenço H.R***., Martin O. and Stützle T. (2003), Iterated Local Search. In Handbook of Metaheuristics, F. Glover and G. Kochenberger, (eds.), Kluwer Academic Publishers, pp. 321-353.
  - Grasas A., Juan, A.A. and ***Lourenço H.R.*** (2014), SimILS: A Simulation-based extension of the Iterated Local Search metaheuristic for Stochastic Combinatorial Optimization, Journal of Simulation doi:10.1057/jos.2014.25

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Hybrid ILS and Extensions

- Hydrid with other metaheuristics
- SimILS
- BiasILS
- MathILS
- Bilevel optimization using ILS
- Other…
  - Parallel ILS
  - Lagrangean Relaxation
  - Column Generation

Word cloud: Blum et al. (20111) Hybrid metaheuristics in combinatorial optimization: A survey. Applied Soft Computing 11:4135-4151

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ILS… SimILS… MathILS…

- ILS represents one of the most efficient yet easy-to-implement frameworks for solving combinatorial optimization problems.
- **Easy to implement and adapt.**
- Most real-life problems are **complex** and filled with **uncertainty**.
- By integrating simulation inside the local search process, SimILS framework extends the virtues of ILS to stochastic COPs as well.
- The same by using exact methods, lower bounds etc…

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Hybrid ILS with other metaheuristics

- Use the ILS structure with other metaheuristics
- **Local Optimization** Phase
  - Tabu Search
  - VNS
  - Simulated Annealing
  - Variable Neighborhood Search
  - ….
- Perturbation Phase
  - Large neighborhood change

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Distribution problem

- Extended Vehicle Routing Problem
  - Heterogeneous fleet (7 different truck capacities)
  - Time windows in the stores
  - Constraints of assigning some trucks to some stores.
  - Maximum driving hours
  - Multitrip for some vehicles
  - Sales constraints
- Minimize operative costs

Grup Alimentari Guissona

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Distribution problem

- GILS-VND Algorithm
  - Coelho V.N., Grasas A., Ramalhinho H., Coelho I.M., Souza M.J.F. (2016), An ILS-based Algorithm to Solve a Large-scale Real Heterogeneous Fleet VRP with Multi-trips and Docking Constraints, European Journal of Operational Research 250 (2): 367–376.
- ILS Structure
  - Initial solution: GRASP
  - Local improvement: Variable Neighborhood Descend
  - Perturbation: Refine using random neighborhood

Distribution problem

- Computational results

Statistical results for the new set of instances: GILS-VND vs. Company.

		GILS-VND			
Instance	Company	Best	Average	Std. dev.	Gap (%)
K	32,472	30,507	30,692	70	−5.48
L	18,997	16,327	16,427	42	−13.53
M	20,266	18,017	18,146	43	−10.46
N	51,609	45,523	45,813	100	−11.23
O	45,764	40,846	40,995	66	−10.42

Distribution problem

- Results
  - Savings 10% daily with respect to the actual solutions.
    - A significant daily amount!
  - Savings of 2% compared with Prins' Algorithm with a simpler version of the problem.
  - Smaller number of vehicles need.
  - Better coordination with sales department.

SimILS

- Stochastic Combinatorial Optimization Problems
  - Uncertainty is present – Random Data
    - Example: Stochastic Demand in Vehicle Routing Problems
    - Stochastic processing times in scheduling
    - etc…
  - Strategic Problems
- Extends ILS to solve Stochastic Models…
- **SimILS**
  - Simulation + Iterated Local Search

SimHeuristics

- Four approaches to simulation optimization:
  - Gradient-based and random search algorithms
  - Evolutionary algorithms and metaheuristics
  - Mathematical programming-based approaches
  - Statistical search techniques.
    - * Fu M C, Andradóttir S, et.al (2000).

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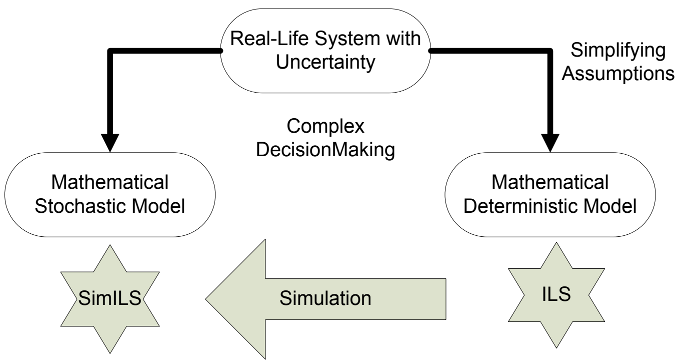
Stochastic Combinatorial Optimization Problems

- SimILS



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SimILS

Procedure *SimILS*

  $s_0 = GenerateInitialSolution$

  $s^* = LocalSearch(s_0)$

  $(s^*, sf(s^*), statistics) = Simulation(s^*, \text{long})$

  Repeat

    $s' = Perturbation(s^*, \text{history})$

    $s'^* = LocalSearch(s')$

    $(s'^*, sf(s'^*), statistics) = Simulation(s'^*, \text{short})$

    $s^* = AcceptanceCriterion(s^*, s'^*, \text{history})$

  Until termination condition met

  $(s^*, sf(s^*), statistics) = Simulation(s^*, \text{long})$

Return $s^*, sf(s^*)$

End

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SimILS
Stochastic Objective Function

Procedure *SimILS*

  $s_0 = GenerateInitialSolution$

  $s^* = LocalSearch(s_0)$

  $ss^* = s^*$

  $(ss^*, sf(ss^*), statistics) = Simulation(ss^*, \text{long})$

  $bsf^* = sf(ss^*)$

  Repeat

    $s' = Perturbation(s^*, \text{history})$

    $s'^* = LocalSearch(s')$

    $s^* = AcceptanceCriterion(s^*, s'^*, \text{history})$

    $(s^*, sf(s^*), statistics) = Simulation(s^*, \text{short})$

    If $sf(s^*) < bsf^*$

      $bsf^* = sf(s^*)$;

      $ss^* = s^*$

  Until termination condition met

  $(ss^*, sf(ss^*), statistics) = Simulation(ss^*, \text{long})$

  $(s^*, sf(s^*), statistics) = Simulation(s^*, \text{long})$

  Return $(ss^*; sf(ss^*))$ and $(s^*; sf(s^*))$

End

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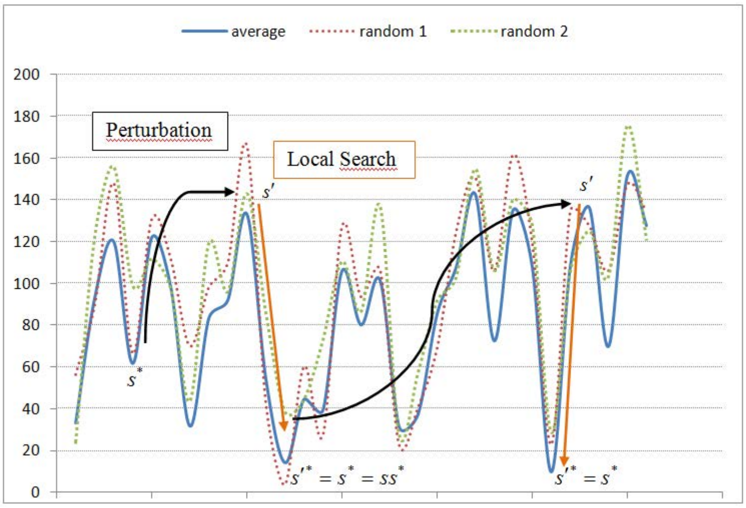
Exemplification of the SimILS for a COP with stochastic objective function



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SimILS
Stochastic Constraints

```
Procedure SimILS
    s0 =GenerateInitialSolution (input data: average values)
    s* =LocalSearch( s0 )
    (s*, sf(s*), service level)=Simulation( s* ,long)
    Repeat
        s' = Perturbation( s* ,history)
        s* = LocalSearch( s')
        (s*, sf(s*), service level)=Simulation( s* ,short)
    Until verifying service level threshold
    Repeat
        s' = Perturbation( s* ,history)
        s'* = LocalSearch( s' )
        (s'*, sf(s'*), service level)=Simulation( s'* ,short)
        s* = AcceptanceCriterion( s* , s'* ,service level, history)
    Until termination condition met
    (s*, sf(s*), service level)=Simulation( s* ,long)
Return (s*; sf(s*))
End
```

Applications

- Permutation Flow-Shop Problem with Stochastic Processing Times
  - Stochastic Objective Function
  - Initial Solution: NEH heuristic by Nawaz et al. (1983).
  - Monte Carlo Simulation
  - ILS method
  - Expected Makespan

  - Juan AA, Barrios BB, Vallada E, Riera D, Jorba J (2014)

Applications

- Inventory Routing Problem with Stochastic Demands and Stock-outs
    * The problem consists in defining a routing distribution plan that includes the product quantities to deliver to a set of retailers.
    * Stochastic objective function (that is, with inventory holding or stockout costs)
    * stochastic constraints
    * The service level is given by a refill policy of the estimated demand.
  - Juan AA, Grasman SE, Caceres-Cruz J, Bektaş T (2014)

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Home care planning

- Home Care (HC) is defined as "medical and paramedical services delivered to patients at home"
  - Health care planning is an important issue today.
  - Service vs Cost
- Take into account:
  - Medical issues
  - Nurses human resources issues
  - Routing and vehicle constraints
  - Period planning
  - Stochastic travel times

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Home care planning

- Decisions
  - Routing (as in VRP)
  - Assign patients to nurses (medical issues)
  - Assign services (patient-nurse) to a day
  - Extra hours limitation
- Minimize total cost
  - Satisfying many constraints related with medical, human resources and other resources.
- Have a collaborative vision!

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Home care planning

- Algorithm SimILS
- Preliminar results of the SimILS show significant improvement with respect of actual solutions.
- Extend to multiobjective stochastic problem.
  - Joint work with Marta Galvani and Federico Malucelli (Politecnico of Milano)

Bias Ramdomization ILS

- Biased Randomization of Heuristics
  - Introduce of a slight modification in the greedy constructive behavior that provides a certain degree of randomness while maintaining the logic behind the heuristic.
  - BRPs can be categorized into two main groups according to how choice probabilities are computed:
    - BRPs using an empirical bias function;
    - BRPs using a skewed probability distribution.

Biased Randomization of Heuristics: supporting real-time decision making in transportation, logistics, and production (2016) A. Grasas, A.A. Juan, J. Faulin, J. de Armas, H. Ramalhinho preprint

Bias Ramdomization ILS

- Skewed Theoretical Probability Distributions

```
Procedure BRP(L, s, PD, p)

•    μ ← using seed s, generate pseudo-random number in [0,1)
•    ρ ← using μ, generate random variate from distribution PD(p)
•    l ← select the ρ-th element of the sorted list L
•    return l

End
```

Figure 3: Pseudocode to select the next element using a skewed distribution.

Biased Randomization of Heuristics: supporting real-time decision making in transportation, logistics, and production (2016) A. Grasas, A.A. Juan, J. Faulin, J. de Armas, H. Ramalhinho preprint

Bias Ramdomization ILS

- MIRHA: Multi-start biased randomization of heuristics with adaptive local search
- Get random solutions
  - Apply a Greedy Classical Heuristics randomized using a bias distribution:
    - The main idea of these heuristics is to select the next step from a list of available movements, usually according to a greedy criterion.
    - we consider non-uniform and nonsymmetric (biased) distributions, e.g.: the geometric distribution or the decreasing triangular distribution.
  - Local Search

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Bias Ramdomization ILS

- MIRHA algorithm structure

Procedure: MIRHA

```
begin
    Initialization:
        inputData ← Data of the instance considered;
        heuristic ← Heuristic choosed;
        prob.Dist ← Distribution probability used to perform the sampling;
        bestSolution ← get a random solution depending of inputData, heuristic and prob.Dist.;
        bestSolution ← adaptiveLocalSearch(bestSolution);
        stop ← false;
    while stop = false do
        solution ← get a random solution depending of inputData, heuristic and prob.Dist.;
        solution ← adaptiveLocalSearch(solution);
        if cT (solution) < cT (bestSolution) then
            bestSolution ← solution;
        end if
        stop ← evaluation stop rule (true or false);
    end while;
    return bestSolution;
end
```

Juan, A. A., Faulin, J., Ferrer, A., Lourenco H.R., and Barros, B. (2013), MIRHA: Multi-start biased randomization of heuristics with adaptive local search for solving non-smooth routing problem. TOP: Volume 21, Issue 1, Pages 109-132.

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Bias Randomization ILS

- Repeat (multistart)
  - Get an **bias random initial solution** x;
    - Heuristic method or a rando
    - local optimization method
  - For a certain number of iterations:
    - **Perturbation Step**
      - method that makes a … n the structure of the solution
    - Small-steps
      - local optimization method, initial solution x'; final solution x''.
    - **Perform an accept/reject test**
- Return the best solution found.

Skewed Theoretical Probability Distributions

Skewed Theoretical Probability Distributions

Skewed Theoretical Probability Distributions

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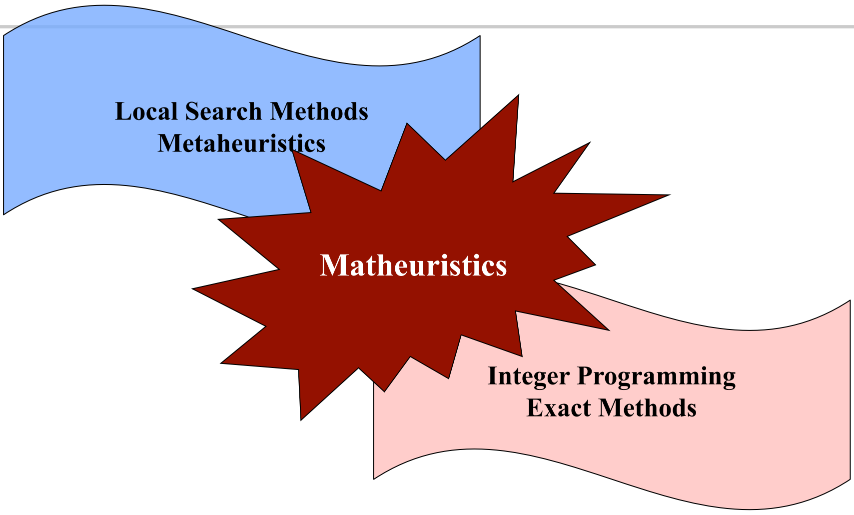
Matheuristics



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Solution Methods

- Tabu Search
- **Iterated Local Search**
- Simulated annealing
- VNS
- Genetic Algorithms
- Evolutionary algorithms
- Ant Colony Optimization
- Scatter Search
- Memetic Algorithms
- Etc… Etc...

ILS 70

Solution Methods

- Branch-and-bound
- Branch-and-cut
- Column generation
- Cutting and price
- Dynamic programming
- Lagrangian relaxation
- Linear relaxation
- Surrogate relaxation
- Etc…

Integer Programming
Exact Methods

Solution Methods

Local Search Methods
Metaheuristics

- Good solutions for complex and large-scale problems
- Short running times
- Easily adapted

- Mathematical proved optimal solutions
- Important information on the characteristics and properties of the problem.

Integer Programming
Exact Methods

Matheuristics

- Refers to …
  - on exploiting mathematical programming (MP) techniques in a (meta)heuristic framework or
  - on granting to mathematical programming approaches the cross-problem robustness and constrained-CPU-time effectiveness which characterize metaheuristics.
    - Copied from **Matheuristics 2010 Conference webpage.**

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Math - Iterated Local Search

- Get an initial solution x;
    - Heuristic method or a random solution.
    - local optimization method
- For a certain number of iterations:
  - **Perturbation Step**
    - **Uses a exact method to solve a subproblem or a relaxation of the problem.**
  - Small-steps
    - local optimization method, initial solution x'; final solution x''.
  - Perform an accept/reject test
    - accept all solutions, accept with a certain probability or accept only if it is a better solution.
    - If x'' is accepted, then x = x''.
- Return the best solution found.

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MathILS

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- Maybe the first application…
  - Use an exact algorithm to solve a sub problem within a Iterated Local Search heuristic for the Job-Shop Scheduling Problem
    - **Solving to optimality the one-machine scheduling problem with due dates and delivery times using the Carlier Algorithm** (1982)
    - Ramalhinho- Lourenço (1995)
    - Lourenço HR. and Zwijnenburg M. (1996), Combining the large-step optimization with tabu-search: application to the job-shop scheduling problem. In Meta-Heuristics: Theory and Applications, I.H. Osman and J.P. Kelly (Eds.), Kluwer Academic Publishers, pp. 219-236.

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Example of Applications

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- Vehicle Routing Problem
  - Iterated Local Search to assign customer to route and optimize the sequencing of the customers.
    - Solve a TSP using Concorde algorithm,
  - Dynamic programming is applied to determine the arriving time at each customer.
    - Ibaraki, Kubo, Masuda, Uno & Yagiura (2001)

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Example of Applications

- Real Applications
  - Maybe the best set of problems to apply Matheuristics methods…
  - Why?
    - * Complex problems with a large number of constraints.
    - * Sometimes difficult to model…
    - * But, a simplification of the problem is frequently a well-studied optimization problem.
  - Apply **metaheuristics for the real general problem**…
  - And **exact methods for the well-known relaxation problem**.

ILS for Bilevel Optimization

- Bilevel Optimization Problems
- The problems has two groups of variables interrelated among them …
  - Strategical decision variables (long term decision)
  - Operational decision variables (short term decisions)
- Hieratical Decision Making
  - Routing Location Problems
  - Production Routing Problems
  - Grouping and Scheduling Problems
  - …

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ILS for Bilevel Optimization

- Get an initial solution x;
  * Heuristic method or a random solution.
  * local optimization method
- For a certain number of iterations:
  - **Perturbation Step**
    * method that makes a large modificatior
    structure of the solution x, resulting in x'.
  - Small-steps
    * **local optimization** method,
  - Perform an accept/reject test
    * accept all solutions, accept with a certain probability or accept only if it is a better solution.
    * If x'' is accepted, then x = x''.
- Return the best solution found.

Optimize or modification on strategical variables

Neighborhood operational variables

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ILS for Bilevel Optimization

- Get an initial solution x;
  * Heuristic method or a random solution.
  * local optimization method
- For a certain number of iterations:
  - **Perturbation Step**
    * method that makes a large modification ba
    structure of the solution x, resulting in x'.
  - Small-steps
    * **local optimization** method
  - Perform an accept/reject test
    * accept all solutions, accept with a certain probability or accept only if it is a better solution.
    * If x'' is accepted, then x = x''.
- Return the best solution found.

Solve to optimality operational variables

Neighborhood on strategical /operational variables

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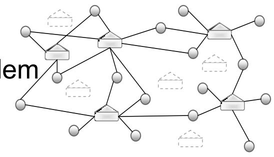
Supply Chain Design for ecommerce



- Supply Chain Design for ecommerce
  - Two-Stage Stochastic Programming Problem
- The goal is to find:
  - the subset of warehouses to be opened;
  - and determine the customer's assignment to the open warehouses
  - … such that all the demand is served at minimum total cost.
  - Demand is stochastic …
  - Each customer area must have 2 or 3 warehouses assigned as regular warehouses.

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Supply Chain Design for ecommerce

- Developed a MathSimILS for this bileve Optimization Problem….
  - Use simulation to obtain the expected overall cost.
  - Local Search on open/close warehouses
  - Exact method to obtain the demand assignment to each simulated scenario.
  - Only *promising* solutions are tested in a stochastic environment.
  - Results compared favorably with Deterministic Equivalent Model solved by CPLEX, in much shorter running times.

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Supply Chain Design for ecommerce

- Deterministic Equivalent Model (DEM)
  - Stochastic Programming

(SCFLPrp) $$\min Z_{stoch} = \sum_{j=1}^{m} f_j y_j + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{s} \pi_k c_{ij} d_{ik} x_{ijk}$$

s.t.

$$\sum_{j=1}^{m} r_{ij} \leq R, \qquad \text{for } i = 1, \dots, n;$$

$$\sum_{j=1}^{m} x_{ijk} = 1, \qquad \text{for } i = 1, \dots, n, \quad k = 1, \dots, s;$$

$$\sum_{i=1}^{n} d_{ik} x_{ijk} \leq q_j y_j, \qquad \text{for } j = 1, \dots, m, \quad k = 1, \dots, s;$$

$$x_{ijk} \leq r_{ij}, \qquad \text{for } i = 1, \dots, n, \quad j = 1, \dots, m, \quad k = 1, \dots, s;$$

$$y_j \in \{0,1\},\ r_{ij} \in \{0,1\}, x_{ijk} \geq 0 \qquad \text{for } j = 1, \dots, m; i = 1, \dots, n; k = 1, \dots, s$$

Open/close Warehouse

Regular/not regular Warehouse

Demand assignment on scenario k

Supply Chain Design for ecommerce

- Some results…
- cap11#,
  - 50 facilities
  - 50 customers
- Capa/b/c#,
  - 100 facilities
  - 1000 customers

	Stochastic Programming			SimILS		
Instance	Z^A_{sim}	t (sec)	gap Cplex (%)	Z^A_{sim}	t (sec)	gap (%)
cap111	879371.7	1103	0	883539.4	170	0.47
cap112	965382.2	3604	0.01	958047.4	303	-0.76
cap113	1047322.2	3603	0.04	1030653.5	294	-1.59
cap114	1160395.4	3604	0.06	1139246.8	50	-1.82
cap124	975397.6	3603	0.09	949375.9	65	-2.67
capa1	-			19244150.7	2263	
capa2	-			18458612.4	2370	
capa3	-			17827599.1	2934	
capa4	-			17162132.3	963	
capb1	-			13773560.0	809	
capb2	-			13378972.7	2645	
capb3	-			13238010.0	2086	
capb4	-			13084859.8	2380	
capc1	-			11704267.9	1363	
capc2	-			11571933.3	1812	
capc3	-			11540950.9	2850	
capc4	-			11535842.8	1928	

Extension…MathSimILS

- Get an initial solution x;
  - * Heuristic method or a random
  - * local optimization method
- For a certain number of iterations:
  - **Perturbation Step**
    - * method that makes a large modification based in optimization and on the structure of the solution x, resulting in x'.
  - Small-steps
    - * **local optimization** method
  - Perform an accept/reject test
    - * accept all solutions, accept with a certain probability or accept only if it is a better solution.
    - * If x'' is accepted, then x = x''.
- Return the best solution found.

Solve to optimality operational variables using Deterministic Equivalent Model (DEM) for Stochastic Programming with a limited number of scenarios

Neighborhood on strategical and operational variables

Other extensions

- Lagrangean Relaxation using ILS to obtain better lower bounds
  - Use ILS to improve the bound obtained by the Lagrangean Heuristics.
  - Work of Martin Gomez Ravetti.
- Multiobjective ILS
- Parallel ILS
  - How to parallelize the ILS?

Metaheuristics

- Which is the best metaheuristic?
  - Begin with a simple method and then turn, if necessary, to a more complicated one or refine the first implementation
  - Small number of parameters
  - Evaluate its performance by:
    - * Accuracy
    - * Speed
    - * Simplicity
    - * Flexibility

Conclusions

- Iterated Local Search
  - Simple
  - Easy to implement
  - Robust
  - Highly effective
  - Modularity
- Start simple and add complexity if needed!
- The success of ILS lies in the biased sampling of the set of local optimal.
- More than 6000 publications in google scholar.

Do you want to try to implement an ILS?

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